(OFF/Sprecherin:)
Diese fötale Position zeigt die Regression in den Mutterleib. Sie hat eine ambivalente Doppel-Bindung erzeugt, weshalb Gödel seit frühester Jugend ältere Frauen verehrt hat, einerseits als Muttererersatz, andererseits um die Mutter auszulöschen. Dies konnte er am besten tun, indem er eine ältere Frau heiratete, welche von der Mutter gehasst wurde.

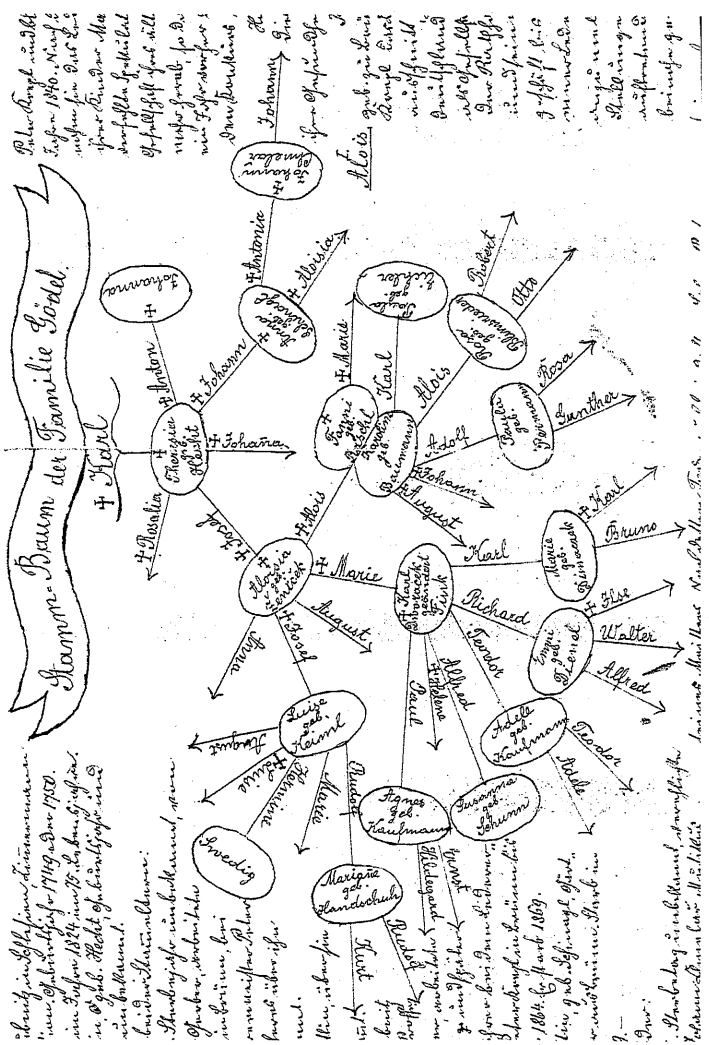
Es gibt jedoch auch Ärzte welche die Meinung vertreten, Nahrungsverweigerung komme bei älteren Menschen des öfteren vor und sei keineswegs eine Besonderheit. Die fötale Position wäre in diesem Fall eine Folge der Magenverkrampfung und des starken Kältegefühls, welches der Patient aufgrund der zu geringen Verbrennung hat.

(INTERVIEW 5 Frau Kahler:)
Es war ein Glück, dass er vorher gestorben ist. Er war absolut verzweifelt, wie sie so schwer krank war, sagt: "Kommen sie meine Frau besuchen!" Sie hat mir mal erzählt: "I have to hold him like a baby."

(OFF/Sprecherin:)
Und damit schließt sich der Kreis unserer biographischen Charakterisierung von Kurt Gödel. Für uns lebt er fort in seinen Werken und Ideen, die wir in den verbleibenden Kapiteln erklären werden.

(Musik: The wheel of fortune)

Shooting Script for the Film:
"Kurt Gödel: A Mathematical Mythos", K42-GöF3
Peter Weibel und Werner DePauli-Schimanovich



K42-GöF3.SW1: Stammbaum der Familie Gödel: Kurt und Rudolf links Mitte am Rand

NSDAP – Gau Wien
Amt für Sippenforschung
Wien, I., Am Hof 3 und 4

Zahl. ... 38

Herr Dr. Gödel (Zuname) Rudolf (Vorname)

Wohnort: Wien VIII Lerchenfelderstr. 81 Stand (led., verh., verw., gesch.): ledig

hat im Sinne der §§ 2 Abs. 2, und 5 der Ersten Verordnung zum Reichsbürgergesetz vom 14. 11. 1935 (Reichsgesetzbl. I S. 1333) den

kleinen Abstammungsnachweis

nach überprüften umseitig ersichtlichen Urkunden erbracht und gilt daher als

Arier – ~~Mischling 2. Grades – Mischling 1. Grades – Jude~~.

Wien, am 1. Dez. 1938

Unterschrift, Amtssiegel
N.S.D.A.P. Amt für Sippenforschung

Undeutlich ausgefüllte Formblätter werden zurückgewiesen!

A 4. Vordr. KR 1/38 – 81.–100. Tausend

K42-GöF3.SW2: Arier-Nachweis von Rudolf Gödel

Magistrat der Stadt Wien
Magistrats-Abteilung 61, Bevölkerungswesen
Wien, I., Rathaus, Stiege 8, Parterre.

M.-Abt. 61 - 103681 1946

~~Zum Amtsgebrauch für~~
Verwaltungsabgabe entrichtet.

Wien, am 26. Nov. 1946

Auszug aus der Heimatrolle

...st Friedrich Gödel

Beruf: Dozent

geboren am 28. IV. 1906 in Brünn

Pfarre (Standesamt) — pol. Bezirk Brünn

Land Č.S.R. , Stand verheir.

ist in der Heimatrolle (/: B.H. VIII – G 188/1937 :/) seit 1937

— in Wien heimatberechtigt eingetragen.

Vater: — Geb.-Jahr: —

Mutter: — geborene — Geb.-Jahr:

Gattin: Adele Thusnelda geborene Porkert Geb.-Jahr: 4. XI. 1899

Kinder: — —

i. A.

M. Abt. 61 – 30 – 46 – 9 – VI/44625 – Janik, Wien

K42-GöF3.SW3: Kurt Gödel war 1946 noch in Wien gemeldet!

D.Zl. 1561 ex 1931/32

PROTOKOLL

aufgenommen in der Dekanatskanzlei der ***philosophischen Fakultät***
der ***Universität in Wien*** *am* Freitag *den* 13. Jänner 1933
um 11 *Uhr* vor- *M*ittags

Vorsitzender:

Dekan: Professor Dr. Heinrich S r b i k

Anwesend:

die Unterzeichneten

Schriftführer: Prof. Menger, Taub

Gegenstand:

Habilitations- K o l l o q u i u m des Dr. Kurt G ö d e l

/ Mathematik /

Form Nr. 90.

K42-GöF3.SW4: Protokoll des Habil-Kolloquiums

Shooting Script for the Film:
"Kurt Gödel: A Mathematical Mythos", K42-GöF3
Peter Weibel und Werner DePauli-Schimanovich

Universidad de Las Palmas de Gran Canaria
DEPARTAMENTO DE INFORMATICA Y SISTEMAS

Campus de Tafira, Apartado 322
E-35080, Las Palmas de G.C., Spain

Werner deViena
DePauli-Schimanovich
Invited Professor

Phone: (+34-28) 45 87 42
Fax: (+34-28) 45 87 11

Spring and Summer in Austria:
Phone: (+43.1) 407.6355-80
Fax: (+43.1) 407.6355-88

Kurt GÖDEL

A Mathematical Mythos

Shooting Script

of the film made by

Peter Weibel and

Werner DePauli-Schimanovich

Translation into English by Eckehart Köhler and Daniela Wolfsberg

ABSTRACT FOR PRESSEDISTRIBUTION

An electronic, experimental documentary film which tries to show (beyond the mere presentation of the biography) new methods for the visualization of abstract trains of thought. A pioneer achievement of electronic media applicable to those intellectual foundations which led to the electronic age. This video film is perhaps the first important statement about our electronic era.

"Gödel, Escher, Bach": this is the title of a cult-book which, surprisingly enough, hit the top of the international bestseller lists. This book deals with the theories of an Austrian mathematician and philosopher (till then only known to the initiated), who probably belongs among the greatest geniuses of this century. But from where does this Kurt Gödel come and what constitutes the essence of this theory? With countless electronic tricks and many examples Dr. Werner Schimanovich and the autor of the film Prof. Peter Weibel try to explain and illustrate the theories and the life of Kurt Gödel in video. The result is, as they themselves call it, a "Mathematical Mystery", a challenging stimulation for the eyes and the mind.

The span of their presentation stretches from Gödel's home Brno, then to Vienna and then to Princeton, USA, where [the friend of Einstein] celebrated several times with honorary doctorates, found the greatest recognition. With his famous proof, Kurt Gödel furnished the mathematical demonstration for the boundlessness of mathematical thought and laid foundations for research in Artificial Intelligence and for computer culture. His fantasy even surpassed that of science-fiction authors, with one difference, namely that Gödel worked on a solid scientific basis. For example he worked out the presuppositions for a time journey into the past.

This year, in 1986, the logician, mathematician and philosopher Kurt Gödel would have reached the age of 80. He meditated on the nature of the relation between man and machine, between mind and mechanism and between brain and computer.

The film by Peter Weibel and Werner Schimanovich describes the life and work of this Austrian, who, according to the judgement of John von Neumann and Sir Karl Popper, brought about the greatest achievement in the field of logic since Aristotle.

PROLOGUE

(Melody: Barcarole)

Kurt Gödel – A Mathematical Mythos
by Peter Weibel and Werner Schimanovich

0. Motto (Linda Koch)
1. The Mystery Gödel (Koch/Harth/Koch)
2. Brünn: Childhood and Youth (Linda Koch)
3. Studies in Vienna (Harald Harth)
4. Vienna Circle (Koch/Harth/Koch)
5. Politics and Science (Harald Harth)

6. Princeton, USA (Linda Koch)
7. Computer Science and Artificial Intelligence (Harald Harth)
8. Turing Machines (Linda Koch)
9. Mathematical Logic (Harald Harth)
10. Window of the Mind (Linda Koch)

(Melody: End of Barcarole)

(Robot/Computer-voice:)
We wish all spectators informative entertainment and entertaining information!

0. MOTTO

(Text OFF/Female speaker)
With respect to mathematics, there is essentially only one rationally supportable point of view.

Namely that of Norbert Wiener, founder of cybernetics: mathematics is a part of our cultural heritage and it is our task to introduce our fellow-men to the secrets of mathematics.

(Text OFF/Speaker)
For a non-mathematician will not readily concede that mathematics is endowed with cultural or aesthetic appeal.

The mathematician as an artist distinguishes himself from the sculptor or musician by the rigid discipline which is necessary in order to become expert in the true nature of mathematics.

(Weibel:)
Whether we want to or not, ...

(Schimanovich:)
Mathematics has long since become a part of our culture, our life;

(Weibel:)
Though invisible for many or visible only in an unfamiliar form.

(Schimanovich:)
In this film we would therefore like to share the viewpoint of Norbert Wiener ...

(Weibel:)
... and try to make a contribution to the aesthetics of mathematics.

1. THE MYSTERY GÖDEL

(Music: Moussorgsky: "Pictures at an Exhibition")

(OFF/Female speaker:)
Who was this man, about whom we know so little and whose work we barely understand? And who nevertheless exerts such a mythical power of attraction?

(Moussorgsky: End)

(OFF/Speaker, deep voice, slowly:)
"Men of such greatness are brought forth by a country only in very great intervals, if at all. If Gödel's discovery had been made in Paris, one would have rung the bells of Notre Dame."

(Sound: Bells of Notre Dame, loudly)

(Bells: End)
(Sound: Noise of a typewriter, first loudly, then in the background)

(OFF/Female speaker:)
This remark was made by a famous French mathematician to Oskar Morgenstern, the great Austrian economic theoretician, who in collaboration with John von Neumann, attained a world-wide fame through their book: "Theory of games and economic behavior". The first initiative to honor Gödel at long last came from abroad. Such a suggestion was made by Professor Morgenstern in a key letter, written in 1965 to the foreign minister of Austria, at that time Dr. Bruno Kreisky:

(Sound: Noise of a typewriter, again briefly in the background)

(OFF/Speaker, deep voice:)
"Without doubt Gödel is the greatest living logician in the world, in fact even scholars as highly ranked as Hermann Weyl and John von Neumann declared that he was unquestionably the greatest logician since Leibniz, if not since Aristotle."

(Sound: Background: Austrian national anthem: "Land of my home, you have great sons ...")

"There is probably no scientist in the history of the University of Vienna whose name outshines that of Gödel. I find it therefore all the more deplorable that Austria has hitherto in no way commemorated its great son. Einstein once said to me that his own work was of little importance any more and that he came to the institute solely for the privilege of walking home with Gödel!"

(OFF/Female speaker:)
Thus professor Morgenstern in his letter to Dr. Kreisky, which, despite Dr. Kreisky's interventions (in the science ministry, (the university and the Academy of Sciences)) nevertheless had no further results.

In 1951 Gödel and the physicist Julian Schwinger jointly received the first Albert Einstein Award.

It was not the mind of an old man with a deeply lined face, to whom we owe the so-called "theorem of the century", but on the contrary, this theorem was the work of an extremely gifted young man of a well-to-do family, who had come into the culturally rich climate of Vienna in the twenties.

Gödel's life and his personality remained anonymous during his whole life. It was his scientific work alone which assumed such mythical proportions and became such a matter of amazement.

It was in 1931, at the age of 25 that this young man published a piece of work under the mysterious title: "On formally undecidable propositions of Principia Mathematica and related systems" which violently upset mathematical thinking, because he showed within the formal system itself that freedom from contradiction of this formal system is not provable. In other words the content of this work says: TRUTH IS LARGER THAN THE POWERS OF PROOF!

Not everything that is true in mathematics can be formally proven valid. Of course this applies only to complex mathematical statements and not to elementary things like the multiplication table. This paradoxical fact, which set a sudden end to a century-long striving, was proved by Gödel with exact mathematical means.

What is the meaning of truth? Of provability in mathematics? At first let us consider the Pythagorean theorem as an example for a true and at the same time also formally provable theorem of mathematics: In a right triangle with the legs a, b and the hypotenuse c, it follows that $a^2+b^2=c^2$.

The truth of this is evident to everyone and is underpinned step by step and argument by argument. We call this contentual proof or verification. This means that we orient ourselves towards certain intuited contents of our thinking. These contents are models which we derive from the relation between objects. Truth is the correspondence (of theorems) with our models. (Its verification is the construction of the correspondence and the refinement of the models.)

If we choose an appropriate triangle, this time not a right triangle, we can ask ourselves what happens if we replace the squares on the three sides of the triangle with cubes. Can we then formulate a similar theorem like the one of Pythagoras, such that $a^3+b^3=c^3$ holds, just as $a^2+b^2=c^2$ held before? The fourth dimension is not representable in an intuitive way. The mathematician nevertheless can formulate the abstract question whether triangles exist on whose sides one can erect four-dimensional "cubes". The four-dimensional cube in this case is a number which is multiplied four times by itself.

At around 1600 Pierre de Fermat further treated the case of arbitrarily many dimensions through an abstract extension of the Pythagorean theorem to the still unproved theorem called the Great Fermat: For every arbitrarily assigned natural number n (greater than 2), do there exist three further positive integers, a, b, and c, such that $a^n+b^n=c^n$ follows? ((Pronounce: a to the n-th power plus b to the n-th gives c to the n-th.))

This Great Fermat lies beyond any geometric intuition. For its proof one needs abstract forms of deduction which in the course of the past 2000 years of mathematical practice have become distilled out as tacit conventions. Later on these forms of conclusion became more explicitly precise and were transformed into a rigid system of rules. With the help of such rules one can move from evident basic truths to higher truths, from these to even more complicated truths and so on. In such a formal system mathematical theorems are treated in the same way as figures in a game of chess.

Since the ancient Greeks one had hoped that mathematical subjects could not only be demonstrated by intuition but also be proved formally in such a system of rigid rules as just mentioned. Even more was demanded: around the turn of the century the predominant current of mathematical research tried to replace the intuitive notion of truth by the formal notion of provability in a system of rules. Up till Gödel's theorem, truth and provability, falsehood and refutability, indefiniteness and undecidability had coincided.

(OFF/Speaker:)
Gödel for the first time showed that there are truths of a standard system of mathematics, the so-called Peano-Arithmetic, which cannot be proved in it. To that extent, truth is more than provability. A present-day game like the party problem illustrates these facts very clearly: How many people at a minimum must I invite to a party, such that, for example, either three of them already know each other from before or three others are not mutually known to each other?

The solution to a generalization of this problem can be proven as true by intuitive combinatorial considerations, but not formally within the standard number-theoretic system of rules. In other words, today we know of concrete mathematical theorems which have these characteristics of which Gödel's theorem speaks: They are true and verified, yet not formally provable. The reason for this, as we know today, lies in the fact that the standard system of rules of the theory of numbers, namely Peano-Arithmetic, is very limited and cannot be adapted or extended as the occasion demands. On the other hand, we can always develop new forms, concerning whose admissibility the community of scientists agrees on by convention.

This thought captures the essence of Gödel's theorem. Luitzen (pronounced: Loitzen) Brouwer, founder of mathematical intuitionism, already advocated this point of view about 60 years ago. In a lecture held in 1928 at the University of Vienna which was attended by Gödel, Brouwer could convince him of his criticism of the limitations of formal language and its methods of proof, and that the contents of mathematics accessible to the mind always exceed the (power of the) formalism. In the course of the historical development of the foundations of mathematics into the technology of computers, a series of other questions arose out of this fundamental

critique of the inadequacy of formal systems, questions as to the relation between language and mind, between machines and thinking, in short questions as to the mechanizability of the mind. For computers are in fact the physical realization of formal systems.

(OFF/Female speaker:)
Gödel's proof therefore raises a comparable restriction for certain current problems of artificial intelligence research. Artificial intelligence is a new science which has become well-known especially through the construction of fast and efficient computers. It has the aim to imitate the mental ability of man with a computer.

(OFF/Child's voice:)
Can machines have a conscience?
Can machines think?
To what extent can machines analize pictures? And scenes?
Can computers understand sentences?

(Electronic noise:)

(OFF/Child's voice:)
Can computers prove arbitrarily designated mathematical truths?

(OFF/Female speaker:)
For all these questions people claim to have found an approach to an answer in Gödel's proof.

For one can appeal to Gödel's proof for the interpretation of philosophical problems, yet formulated in the garb of mathematical rigor. After all, Gödel's proof is a more precise instrument than the usual philosophical reflection for the treatment of abstract speculative questions.

(OFF/Speaker:)
One stands at the head of the path to the ultimate things of the universe, and with Gödel's proof one has a key to overcome the barrier of understanding. From this comes the mythical attraction of Gödel's theorem.

(OFF/Female speaker:)
This track has been followed by a series of works which have been published the last few years which have made Gödel become so popular. In 1979, about a year after Gödel's death, a book came out in America whose author received the Pulitzer Prize a short time afterwards.

(Bach: Ricercar)

At this point Gödel was fairly unknown in Europe. Yet after the publication of the German translation and this sensational success, Gödel was also taken seriously in German-speaking countries and eventually also in Vienna.

This book is a perfect example for our thesis about Gödel's attraction, because it shows precisely how Gödel's mathematical theory together with achievements in music and pictorial arts can come to be worked up into a general theory of knowledge and creative activity.

(Ricercar: End)

As the subtitle of the book says, the central thesis of the author is to see the intellectual productions of man as "an eternal braid", namely thinking about thinking about thinking ...
(Voice of female speaker grows fainter and dies away!)

(OFF/Female speaker:)
The presentation of Gödel's achievements in the field of logic and computer science will be dealt with more fully later on. Here we once again point out that the incompleteness of logic demonstrated by Gödel (namely that there are mathematical truths which one cannot prove formally) show up the limits to which computers are subject in principle.

(12-Tone-Music: Alban Berg)

These theoretical limits however are not yet relevant to present practice, as the current computers are very limited in their practical possibilities in any case. Furthermore Gödel contributed to set theory, the most fundamental theory of mathematica, one of the most celebrated results of the 20th century.

(Alban Berg: End)

(OFF/Speaker:)
Stimulated at an early age by his Viennese teacher Hans Thirring and motivated in particular by his friendship with Albert Einstein, Gödel in the forties and fifties became occupied with cosmology. He calculated a solution to Einstein's field equations of the general theory of relativity.

In Gödel's solution the universe rotates with constant angular velocity around a fixed system of coordinates. The vertical lines of this space-time diagram in Gödel's model of the universe are the geodetic lines through space-time of the principal mass-points of the universe, such as stars and galaxies. These objects determine the global structure of space and time, for which Gödel formulated his extremely surprising solution. On account of the stars and galaxies which generate the world-lines, we call these lines matter-lines. The diagram shows the rotational symmetry around the central matter-line PQ, as well as the vertical translational symmetry. The light-cones, representing the horizons of light signals sent from a given point of space-time, emanate from all points.

What are light-cones? Light spreads out in particles called photons which have no mass. The flash of an explosion as the origin of light creates a cone of light, the generation of which

represents the history of individual photons from the flash. As all particles with mass move more slowly than the velocity of light, their lines, the matter-lines, lie inside the cone.

The matter-lines have a temporal order. A future oriented time-like curve passes from P to Q. A normal time-traveller starts at P and travels along the matter-line to Q. Going the other way, however, it is more difficult to figure out how to arrive at P starting from Q.
(Background noise: Start of a rocket)

The closed time-like curves in Gödel's universe, however, have the property that the time-traveler can start at any space-time-point on them and return back to it.
In Gödel's universe therefore a voyage into the past is possible if the time-traveler moves out of his local horizon. This would of course only be possible with gigantically large rockets, where one would practically have to incinerate whole planets.

If the world-line deviates far enough, farther than the distance to the (theoretically possible) optical horizon, that is, beyond a certain critical radius, then closed world-lines become possible. The world-lines return to the same space time point, from where they departed.

Such a trip though time represents an old dream of mankind which is also expressed in the science-fiction novel "The Time Machine" by H. G. Wells.
"The Return of The Time Machine" by Egon Friedell, 1946, a parody on H. G. Wells, deals with voyages into the past and the problems raised by them. Egon Friedell was one of Gödel's favorite actors in the Theater in der Josefstadt during his college years.

(Electronic music)

Globally seen, that is, for distances greater than the optical horizon, a voyage into the past naturally overturns the causal structure of the universe.
For this reason, Gödel's solution brought about a discussion about large-scale causal structures in physics which lead to a more precise explication of the causal effect of an event. Of course, interference in a causal chain is only possible if the result does not destroy its own cause. In this sense one could say: trips though time take place outside of causality. It therefore seems that time-travelers must be spirits! Perhaps it was this which led Gödel during the last years of his life to become interested in the literature on ghosts and demonology, as this extract from his booklist shows.

His model stirred up a lot of excitement in the fifties and its influence can be traced up to Steven Spielberg's film "Back To The Future", whose action sequences, however, must be considered absurd from the physical point of view.
It is clear that Gödel's universe with the possibility of traveling backwards in time is a mere theoretical model. We would nevertheless now like to undertake a journey into the past by film and photography into Gödel's unknown life, back to the place of his birth.

2. BRÜNN: CHILDHOOD AND YOUTH

(Music: "When Bohemia was still a part of Austria ..." / "Wie Böhmen noch bei Österreich war ...")

(OFF/Female speaker:)
Gödel spent his childhood and youth in Brünn, the present Brno. Around the turn of the century Brünn was a predominantly German-speaking city in the Czech part of the Austro-Hungarian monarchy and the capital of Moravia. Today Brünn is the second largest city of Czechoslovakia. Moravia and Bohemia (with its capital Prague) have a great intellectual tradition.

(Music: End)

From the mysticism of Jakob Böhme to the rigorous analytics of Ernst Mach, the forefather of the empiricist philosophy of science of the Vienna Circle.

From Jan Amos Comenius, the prophet of the world as a labyrinth, to Franz Kafka's empiricism of alienation; of the Golem, the first artificial man, to Gustav Meyrink, his literary resurector, we find jewish-cabalistic traditions full of mysticism and of clear rationality (all contemporaneous to each other).

Famous personalities directly from Brünn and its surroundings include: Gregor Mendel, discoverer of the fundamental genetic laws, and the architects Josef Hoffmann and Adolf Loos.

Whoever believes in the importance of cultural environment for the development of talent can find in this tradition (of minute analysis combined with mystical longing to strive beyond human limits) the ideal preconditioning for the intellectual development of the young Gödel. The rich cultural climate was made possible by the industrial expansion (prosperity), especially of the textile industry which was chiefly in German hands.

(Music: Bohemian Polka)

The prosperous industrialists lived in German-speaking residential suburbs where art nouveau was a favorite style of architecture. Like all booming cities, Brünn also had its dark sides: slums where cheap laborers and domestic servants lived, mainly of Czech nationality.

Into Brünn at the turn of the century, Gustav Handschuh, grandfather of Kurt Gödel, had moved, coming from the Rheinland. (Handschuh means glove in German.) He made his career in the textile industry in the firm Schoeller. His wife, Rosita Bartl with maiden name, came from the German-speaking language-enclave of Iglau in Moravia and was very tradition-conscious. The Handschuh grandparents lived in Bäckergasse 9, on the second floor of a typical house in a plain style with interior courtyard and open corridors, where the neighbors met in the evenings to chat. Their daughter Marianne Handschuh, the later mother of Kurt Gödel, grew up here.

Also in the same house, on the first floor, lived Kurt Gödel's father Rudolf, together with his foster mother Aunt Anna. He was also born in Brünn. (Gödel means godfather (at confirmation ceremony) in German.)

(Bohemian Polka: End)

The parents of Rudolf Gödel senior came from Vienna and resided mainly there. After the death of the senior Rudolf Gödel's father, Rudolf was taken on by Aunt Anna.

The Handschuh grandparents had very good contact with Aunt Anna. Frequently the played music together or played theater. In this way Marianne and Rudolf got to know each other very early and married later on.

Soon the couple had an apartment in the Heinrich Gomperzgasse 15. In 1902 their first son was born, who was also named Rudolf.

Then the couple moved back to Bäckergasse, to the street of their childhood, right next to house number 9, where the grandparents Handschuh and Aunt Anna lived.

(Music: "Spring Voices" by Johann Strauss)

In this house, on April 28th, 1906, unnoticed by the world, the genius mathematicus Kurt Friedrich Gödel first saw the light of day. The children would play in the garden behind the house.

At the age of eight Kurt suffered severe attacks of rheumatic fever: the beginning of a lifelong hypochondria was marked off.

Before the First World War, the Gödel family moved from the Bäckergasse to their own villa in the Spielberggasse 8a which was, as this panorama shows, in the immediate neighborhood of the former apartment.

(End Music: "Spring Voices")

Just as grandfather Handschuh, Rudolf Gödel senior had made his career in the textile industry of the Redlich factory in the Straussengasse, and had there become a partner and factory manager. The income of a factory director afforded him a high standard of living. For example, the family drove one of the first Chryslers in Czechoslovakia; also the household furnishings were of high quality. The villa came with an extensive garden with a summer house on the south slope of the Spielberg. With a telescope the children could make out the stone ornaments on the gothic cathedral of St. Peter and Paul. From the windows of the villa they could view the notorious prison on top of the Spielberg.

Kurt was a very talented and industrious pupil in high school, without any disturbing problems of puberty. At the age of 14 his interest in mathematics was roused, at 16 he already studied Kant.

From the first grade of elementary school up to the end of secondary school he had almost nothing but best grades (all "A"s).

(Interview 1 Brother, ((Med. Rat)) Dr. Rudolf Gödel:)
((My brother was of course excellent in mathematics, but also in Latin and it was rumored that in all his Latin assignment for seven years he had not a single grammar mistake.))

(Interview 2 Brother:)
On Sundays we usually made an excursion to the beautiful surroundings of Brno. And he normally stayed at home with a book and didn't want to come along and join us. And that did not please my father very much.

(Interview 3 Brother)
After the war we were in Marienbad several times with my brother and I remember that we read Goethe together, the biography of Chamberlain. And he got very interested in his Theory of color and maybe that was the reason that he turned his interest toward the natural sciences.

(Interview 4 Brother:)
Especially after World War I, together with him as well as with my mother, I read many biographies which came out after World War I. Including some about the Habsburgs, and some about (the) great politicians of the time. ((We would debate about our readings, also with my mother, who was very intelligent and who liked to read these things.))

(OFF/Female speaker:)
Sunken in his books, he seemed oblivious to his cultural surroundings. Instead of Prague as place for his studies he chose Vienna.

(Sound of steam train)

3. STUDIES IN VIENNA

(Music: "Vienna, Vienna, only you alone ..." / "Wien, Wien nur du allein ...")

(OFF/Female speaker:)
In 1920 Rudolf Gödel went to Vienna to study medicine. Four years later in the Fall of 1924, Kurt joined him. Rudolf went to pick him up at the Viennese Northern Railroad Station and brought him to his apartment.

The first apartment where Kurt lived together with Rudolf until April 1927 was to be found in Florianigasse number 42. The very gifted young man, from a well-off family that assured him trouble-free studies, arrived into an extraordinarily prolific cultural milieu.

(End: "Wien, Wien, nur du allein ...")

(Interview 1 Brother:)
All of my teachers and examiners were people of world-wide renown: the internist Wenckebach, the surgeon Eiselsberg, the pediatrician Birkee, the neurologist Wagner-Jauregg, in fact each man greater than the next.

(OFF/Speaker:)
At first the young Gödel signed up for mathematics, physics and philosophy. Originally he wanted a degree in physics. Gödel studied under Hans Thirring, who held his lectures in the auditorium for theoretical physics, which is situated on the fifth floor in the Strudelhofgasse number 4. The Mathematical Institute was also in the same building. However, not until two years later, in 1926, did Gödel decide to make mathematics his major. His teachers at the Mathematical Institute were:

Hans Hahn, the "actual founder" of the Vienna Circle,
Karl Menger, the son of the famous economist Carl Menger,
Philipp Furtwängler, the cousin of the conductor, and
Wilhelm Wirtinger, the algebraist.

His most influencial philosophy teachers were Moritz Schlick and Heinrich Gomperz, and later also Rudolf Carnap (the logician and philosopher of science.)

They held their lectures in the University of Vienna's Main Building and several were also members of the Vienna Circle, whose meetings (in the same building as the Mathematical Institute) Gödel began attending in 1926.

(OFF/Female speaker:)
Between the years 1924 and 1929, Gödel changed his living quarters relatively often.

(Interview 2 Brother:)
Well, in society he was actually rather cheerful and friendly, he enjoyed telling and hearing jokes. But he never talked about actual events about his work for instance. When he came to university he already knew the stuff, because he studied it in highschool.

(OFF/Female speaker:)
As a student he led an active life. In his circle he rapidly earned a reputation as being an extraordinary talent. From October 1927 until June 1928 Gödel lived in Währingerstrasse number 33 in the house of the Cafe "Josefinum", which he often frequented with his Vienna Circle colleagues.

(Music: "Vienna Blood" / "Wiener Blut")

In the period from July 1928 until November 1929, Gödel lived in a large apartment at Langegasse number 72 which was originally planned to be his parent's living quarters. Diagonally opposite from this apartment, in Langegasse number 65, lived the danseuse Adele

Porkert, who later became his wife. At this time she was still married to the photographer Nimbursky and made appearances in the nightclub "Nachtfalter" downtown on Petersplatz 1.

(End: "Vienna Blood")

During the time he was working on his dissertation, Kurt Gödel probably met the attractive Adele for the first time in the Langegasse. In his dissertation he quickly solved the problem of the completeness of predicate logic, which David Hilbert, the greatest mathematician of the time and his assistant Wilhelm Ackermann in 1928 presented in their textbook "Elements of Theoretical Logic". This book is still today considered a standard text.

(Voice over/Valie Export:)
The formulation of the problem was: "Whether the Axiom system ist complete at least in the sense that really all the logically valid formulas can be derived from it, is still an open question."

(OFF/Speaker:)
After the death of Gödel's father, his mother moved to Vienna. Therefore a bigger apartment was needed, and they moved at the beginning for November 1929 to Josefstädterstrasse 43. This flat remained the Gödel family's main residence until November 1937 and is the residence perhaps best known to the scientific community.

Here, in the period between 1929 and 1937, he worked on his celebrated writings and corresponded with mathematicians from all over the world: Ernst Zemelo in Freiburg, Jacques Herbrand in Paris, Oswald Veblen in Princeton, Paul Bernays in Göttingen and John von Neumann, the citizen of the world.

The Gödel family, especially under the guidance of their cultivated mother, regularly engaged in civic cultural activities. Mother and sons had season tickets for the nearby Theater in der Josefstadt in their favorite box.

(Background music: Maria Cebotari)

In music he especially loved Italian opera and Viennese operetta, particularly Johann Strauss and Richard Strauss. He disliked Johann Sebastian Bach and Richard Wagner.

The songs that he liked to listen to were from the "light" repertoire of the time: Schubert's "Am Brunnen vor dem Tore", "Brüderlein Fein", the "Barcarole" from "The Tales of Hoffmann", or Spanish dance music.

(Cebutari: End)

(Background: Wheel of fortune)

However he was also interested in American hits like: "Harbour Lights", "The Wheel of Fortune", Eddie Fischer's "O mein Papa" and later especially American Pop-music.

In the visual arts he tended towards surrealism and abstract art. Later on in New York he often visited the Museum of Modern Art.

(Background music: End)

Already during his studies, starting in 1926, Gödel was invited to join the Vienna Circle.

(Interview 1 Prof. Edmund Hlawka, Math. Inst.:)
He is of course very strongly influenced by Hans Hahn and Karl Menger. He also attended their lectures on set theory and real functions.

(OFF/Speaker:)
In October 1929 Gödel began to regularly visit Menger's colloquium. Karl Menger, the son of the economist Carl Menger, regularly held mathematical colloquia. This took place in the mechanical drawing room of the Institute of Mathematics. As an exceptionally active participant of the colloquium, Gödel had at this period his most lively and zestful time. He met there many specialists and colleagues like Alfred Tarski and John von Neumann. The editor Karl Menger soon made Gödel and later Franz Alt Assistant Editor of "Results of a Mathematical Colloquium". Gödel himself also published unusually much in this period: 13 publications during the years 1929 – 1937. ((A year after his dissertation of October 1929, he discovered in 1930 his theory about formally undecidable sentences.))

(Interview 2 Hlawka:)
((... also attended the lectures on number theory of Furtwängler. And that was, I believe, the stimulus for him to apply the methods of number theory to logic. I mean the correlation of the sentences of logic and mathematics with natural numbers which one nowadays calls Gödelization.))

4. THE VIENNA CIRCLE

(Music: "The Blue Danube" / "Donauwalzer")

(OFF/Female speaker:)
In the years from 1900 until 1930 there was in Vienna an unparalleled flourishing of culture: a golden pentagram of physics, philosophy, medicine, psychology and economics which formed the threshold of the 20th century. A wealth of geniuses. There were many scientific and philosophical circles which met regularly. The one that later entered into history as the "Vienna Circle", was the Schlick-Circle, named after its central figure, Moritz Schlick.

This circle entered into the public eye in 1929 with the publication of the manifesto "The Scientific World-View: The Vienna Circle". This was dedicated to Moritz Schlick and signed by Hans Hahn, Otto Neurath and Rudolf Carnap.

(End: "Blue Danube")

Hans Hahn, professor of mathematics, was the actual founder of the Vienna Circle. In 1922 he arranged the appointment of Moritz Schlick at Vienna. Thus Hahn was able, together with his earlier student colleagues Philipp Frank and Otto Neurath, to act out his old dram of founding a creative cell for encouraging scientific advancements (in philosophy) in Vienna.

Otto Neurath was the organizer of the Vienna Circle and at the same time leader of the leftist fraction, interested in social reforms, who regularly held meetings at the "Volksheim Ottakring", (a famous adult-education center in a blue-collar district of Vienna.)

(Background music: "Internationale")
Rudolf Carnap was the most radical philosopher and became after his emigration, (even more than before), the best-known representative of the Vienna Circle in America.

Between the years 1924 and 1933, the meetings of the Circle were conducted regularly at 6 p.m. These meetings were ...

(Interview with Viktor Kraft:)
"... held every other Thursday in the Mathematics Institute because a Philosophy Institute did not yet exist at the time."

(OFF/Female speaker:)
Today in this part of the building is located the Institute of Meteorology.

(Interview Herbert Feigl:)
The Vienna Circle consisted of a group of scientists interested in philosophy and philosophers (whose studies got them) interested in science.

(OFF/Female speaker:)
Other than Hahn, Neurath and Carnap, central figures of the Vienna Circle, there were Moritz Schlick, Karl Menger, Herbert Feigl and Friedrich Waismann, Viktor Kraft, Felix Kaufmann. Closely connected were also Ludwig Wittgenstein and Sir Karl Popper, as well as the Berlin Group around Hans Reichenbach. They all testify to the fact that the Vienna Circle would become a world-wide movement.

(End: "Internationale")

(OFF/Speaker:)
Gödel was a member of the Vienna Circle since 1926. As the first publication testified, the Vienna Circle attempted to carry on the analytical tradition of Ernst Mach, who taught in Vienna since 1895 and exerted a great influence in art and culture. (Mach also became a hero of the working class in Vienna. So for this reason Otto Neurath established the Ernst Mach Society (which members of the Vienna Circle organized.))

Mach had taught that the conceptual analysis of contradictions among theories leads to progress in our knowledge of the world.

(Noise of an airplane, briefly)

The Mach-number, named after Ernst Mach, states the ratio between the speed of an object and the speed of sound. With Mach 1 an object flies at the speed of sound. The Concorde flies at Mach 2.

Ludwig Boltzmann applied the concept of entropy of thermodynamics to the energy of moving atoms in a gas. This new interpretation of the second law implied that a system in a state of lower entropy can always only pass into a state of higher entropy. (He showed, in his epoch-making atomic theory of heat, that the central concept of entropy can be interpreted in terms of the average energy of moving atoms.)

Boltzmann, as advocate of atoms, would become significant for the Vienna Circle for his concept of the model, namely for the thesis that our scientific knowledge grasped not nature itself, but only a model of nature. These models change according to our theories and laws and must be:

(1) logically consistent,
(2) empirically correct, which means corresponding with experimental facts,
(3) relating a maximal amount of information about the facts, and
(4) economically organize facts, that is, be minimally redundant.

This precise conception of a theory of models already plays a great role in quantum physics and today in the database conception in computer science. Mach and Boltzmann were, among other things by their abandonment of absolute concepts, forerunners of Einstein's theory of relativitiy.

Of his many visits to Austria, Einstein's lecture (in 1921) in the Künstlerhaus before an audience of 3000 is especially worthy of mention. Gödel first met Einstein in person in the fall of 1933 at Princeton in the USA.

Gödel had many friends in Vienna, with whom he had a great number of enlightening discussions. He used their inspiration in his famous proof: In the biggest center of Mathematics at the time, namely Göttingen in Germany (under the leadership of David Hilbert), language was merely seen as a formally mechanized system. On the other hand, in the Vienna Circle, Gödel would soon learn of the language as a philosophical tool in the sense of a linguistic critique of science. Wittgenstein's "Tractatus logico-philosophicus" which presented a mathematical-logical interpretation of language, played a leading role in the discussions of the Vienna Circle and was often read and interpreted in the group.

(Interview 1 Heinz Zemanek:)
He had with his early philosophy the idea that one can perfectly describe the world in sentences which are either clearly false or clearly true. That is precisely the world of the computer, in which all processes consist of small elements that either hold true or not ((which from the standpoint of logic are either true or false.))

(OFF/Female speaker:)
Wittgenstein's philosophy of language was stimulated by Heinrich Hertz, Gottlob Frege and Bertrand Russel, but most of all by the Old Austrian Fritz Mauthner. Mauthner came from the German part of Bohemia, studied in Prague and later lived in Berlin.

He corresponded with Ernst Mach and was (as a result of his work) published in 1900 – 1902 and entitled "Critique of Language" ("Kritik der Sprache", that is), the founder of a linguistic philosophy.

Fritz Mauthner had already equated thought with use of language, but this could not be upheld in its original formulation. Wittgenstein and Carnap attempted to find a better scientific foundation for this standpoint. The early Wittgenstein wanted his principle "The bounds of my language are the bounds of my world", to apply also to mathematics. Which means mathematics can only treat of such objects which the language of mathematics can formulate. Implied here is the possibility of the discovery of mathematical facts provided by means of the language of mathematics. (The formalized language of mathematics is predicate logic together with its proof techniques. This logic originated through a schematization of language in the sense of structural linguistics. David Hilbert also wanted to formalize mathematics with predicate logic.)

Gödel's famous discovery not only consists in disproving Hilbert's Program, but also Wittgenstein's early principle. Gödel demonstrated that the bounds of language do not amount to the bounds of our world. Wittgenstein later also revised his standpoint (by recognizing Gödel's method as a "new kind of proof").

(Interview 2 Heinz Zemanek:)
However we know that later in his life Wittgenstein saw that this ideally perfect description of the world did not prove correct. And the tension that we can perceive in both of Wittgenstein's philosophies, between the logical order of the early philosophy and the living reality of the late philosophy, exists as a matter of fact in every computer application.

(OFF/Female speaker:)
Contrary to the early Wittgenstein and to Mauthner, Gödel realized that the language of mathematics and its formal proof systems are incomplete. Stimulated by the linguistic approach of the Vienna Circle and Brouwer's intuitionism, he was, after Paul Finsler, the first to perceive that certain mathematical sentences are undecidable, and moreover he could prove as much:

The mathematical world proved to be stronger than its language. Language revealed itself to be weaker than thought.

That which can be proven in and by language is less than that which thought can perceive to be true, and that is in turn less than what is possible in the world.

(Background music: Blue Danube Walz)

Gödel shared his discoveries with his friends Rudolf Carnap and Herbert Feigl in the "Cafe Reichsrat" for the first time in August 1930. His work was cited as the theorem of the century and had revolutionized the world view of mathematics fifty years ago. It destroyed the holy temple of a mathematics which was supposed to be representable in one complete and consistent system. Today we call Gödel's famous accomplishment simply "Gödel's proof".

(End: Blue Danube)

(Interview 1 Hlawka:)
In his dissertation he showed that mathematics is complete, in his habilitation, that it is incomplete: A joke among logicians.

(OFF/Female speaker:)
As one consequence of our presentation it may be said that Gödel's work was a result of the efforts of the Vienna Circle, without whose environment it would appear unimaginable that Gödel's work could come about.

However, after the hard work Gödel had to withdraw to the Purkersdorf Sanatorium, built by Josef Hoffmann. ((The two years of intense concentration for his dissertation and habilitation brought about a serious nervous crisis for his health, unstable as it was in any case, from the end of 1931 to the beginning of 1932. The exhaustion was so great that Gödel had suicidal tendencies, and his family was very worried about him.))

(Interview 1 Rudolf Gödel:)
((Well, yes)) it was really a kind of depressive condition and feelings of anxiety that occurred for the first time shortly after his well-known work was published. Today one would describe it as an endogenous depression. At that time such a term ((and such a diagnosis)) did not yet exist. ((And then at times such conditions were repeated. He was very cheerful at times and felt fine, and then again very depressed, when he just had to go to a sanatorium for a time. Let's say that because of his weak nerves, he was twice in sanatoriums. It was at least a few weeks in any case. Purkersdorf and Rekawinkel.)) I only know that for a while we were afraid that he would commit suicide. That is the reason that we brought him to a sanatorium.

(Interview 2 Hlawka:)
He was often occupied in the library reading books, but often he kept still, or looked at the same page several times, or for a long time. But he was inaccessible or he looked very reserved to us. In October 1934, at the time I was in my first semester, at this time he was already well-known because of the theorem that he proved in his habilitation, that every formal system is incomplete.

(Interview 2 Brother:)
((Professor Menger came at that time to my mother (my brother was by chance not at home) and said to my mother: "Do you know that your son is a celebrity?" And in this way we learned

that my brother was indeed an important man, for he did not like to speak about his profession and certainly not at all about his distinctions or titles and such things.))

(OFF/Female speaker:)
((Only at a period of the highest concentration of scientific ability in a single place, Vienna at the time of the twenties, could such a masterpiece succeed. Gödel had rationally formalized the labyrinth of human thought for the first time, where human thought itself becomes a labyrinth.))

5. POLITICS AND SCIENCE

(Sound: artillery, machine guns, marching troops)

(OFF/Speaker:)
In the thirties the upheavals in Austria began to reveal the true face of Austrian politics and to bring life about the downfall of this scientific and cultural flowering. For quite some time already, Austrian society between the wars was merely a facade, which, to be sure, still had its architectonic flourish and in its design exhibited a great style in its beautiful appearance, but in its core it was already decaydent and hollow. Soon this facade of culture broke to pieces. Hitler needed only to sweep them up.

(Background music: "The Banner high" / "Die Fahne hoch")

Austrian Fascism and German National Socialism appear to us today as a campaign of old powers against the new world of rational science. Feeling and belief stand against reason and analysis. Metaphysics and parochialism revolted against the coming age of the technologizing of our world to which we are more indebted for the improvement in our present day living conditions than to any obsolete ideology.

Already since 1928, the majority of the Austrian students had strong pro-German nationalistic convictions and increasingly created disturbances at gatherings of Jewish, socialist or liberal professors.

(End: "The Banner high" / "Die Fahne hoch")

The attacks especially on the Vienna Circle became more and more violent. It culminated when Moritz Schlick, who was considered a Jew, although in reality descended from old German nobility, was shot on the 22nd June, 1936 on the staircase of the Philosophical Faculty in the University of Vienna.

His former student, Hans Nelböck, spurred on by public opinion as well as by the views of his fellow student Leo Gabriel, was the one responsible. He was, incidentally, released from jail soon after Hitler's occupation of Austria.

(Interview 1 Rudolf Gödel:)
Well for example that was a time when his nerves were not in very good shape. He liked Schlick very much.

(OFF/Speaker:)
For years blacklists circulated among the national-socialist students, to whom Erich Heintel also belonged, and who has now been for many years after the war a Philosophy Professor (at the University of Vienna.) The lists carried the names of Jewish scholars and those connected to or sympathetic towards them.

Gödel's name was on such a list because he was a student of the Half-Jewish Hans Hahn and a member of the Vienna Circle which was rejected as Jewish in its philosophy; but actually he possessed a "solid" Arian descent. However mathematical logic and set theory were condemned as "Jewish", just as was Einstein's theory of relativity. For this reason, Gödel was attacked near the Institute of Mathematics at the Strudelhofstiege at the beginning of November 1939 by right-wing students ((in SA-uniform, the same SA-uniform in which Erich Heintel lectured in the Auditorium Maximum)).

After his habilitation under Hans Hahn ((with the co-assessment of Wirtinger, Thirring, Menger and others, Gödel became Privatdozent in 1932, which corresponds in rank to assistant professor)). Though unsalaried, he taught between 1933 and 1938 at the Institute of Mathematics.

(Interview Hlawka:)
... furthermore his face was turned towards the blackboard, not towards the audience. The lecture hall was nevertheless overcrowded (in the small lecture room, where the memorial plaque for Gödel hangs), but I have to say that in the next meetings the lecture room got fairly empty.

(OFF/Speaker:)
The newly established Institute for Advanced Study in Princeton, which was at the time still situated in the old Fine Hall of Princeton University, and to which Albert Einstein also belonged, had recognized the significance of Gödel's discovery very quickly and had invited him to Princeton. Gödel traveled to America for the first time in the Fall of 1933 and taught in Princeton from February until May of 1934.

Through his lectures at Princeton he contributed to the establishment of the American school of logic, especially to the further development of metamathematics and the theory of recursive functions by Stephen Cole Kleene. Also the founder of the lamda-calculus, Alonzo Church, had applied the incompleteness-result in his formal system. Gödel's proof was further strengthened by Barkley Rosser (shown at far left in the picture). After a year's stay back in Vienna, Gödel went in 1935 back to America, but he had to immediately return to Vienna for reasons of health.

(Interview 2 Rudolf Gödel:)
((He once stopped over in Paris on his way back from America, and called me to say that he did not trust himself to ride (at that time with the train) from Paris to Vienna, and I then went to Paris. He had an hour-long telephone conversation with me, and I then traveled there to pick him up.))

(Background melody: "In the suburb Grinzing (of Vienna) there is a street called "street to heaven"; that cannot be by chance! God the Lord had let grow there a heavenly wine." / "In Grinzing gibt's a Himmelstraßn, das kann ka Zufall sein. Der Herrgott hat dort wachsen lassen, an himmlisch guatn Wein.")

(OFF/Speaker:)
(In November 1937 Gödel moved to Grinzing (which is the famous Viennese wine garden district) for two years ((in a studio apartment at Himmelstraße 43 where the conductor Karl Böhm later lived.)) On September 20th 1938 then married Adele Nimbursky after knowing her for ten years.

(Interview 3 Brother:)
Well, one was not in complete agreement with this choice. Of course, she was not a match for him intellectually, but this would lie in the nature of things. But she did come from a really simple background. (Her parents also lived in the Langegasse. Her father was a photographer, he had his own photography studio.)

(Background: "We are the fighters of the National-Socialist-Democratic-Labour-Party" / "Wir sind die Kämpfer der NSDAP")

(OFF/Speaker:)
Already two weeks after the marriage, Kurt Gödel once again left his wife and Europe (from Cuxhafen) on board of the "New York". Adele stayed behind in Vienna alone. During Gödel's next stay in Princeton in the Fall term '38/'39. Meanwhile the Nazis as the new rulers of Austria, stripped him of his instructorship position. This order by the Dean of the Philosophy Faculty, Professor von Christian, was supported by the lecturers union leader Dr. Marchet. Gödel astonishingly applied for a similar position of the new order, which was finally approved a year later by the German Reich ministry in Berlin.

At the same time, Gödel was called up for the draft by the German army and at the mustering examination was found fully fit for the front. In extreme distress, he applied directly to Oswald Veblen, who was a Professor at the Institute for Advanced Study at Princeton, and with his help succeeded in getting himself and his wife emigration visas to the USA. For various reasons he subsequently turned his back on an Europe that was to fall prey to destruction.

(End: "Fighters" / "Kämpfer")

(Music: "For we are campaigning against England" / "Denn wir fahren gegen Engeland")

(Interview 4 Rudolf Gödel:)
It was probably both things: The aversion at that time was naturally against Nazi-Austria, that was for sure. And then the prospects for him in America were apparently much better, since (after having been guest professor there two or three times) he was made an offer right away there. He had a wonderful institute, including splendid rooms looking out onto the woods) and probably a large salary. Therefore it was for him of course a situation which he would never be able to have in Vienna.

(End: "Because we are campaigning ..." / "Denn wir fahren ...")

(OFF/Speaker:)
Because of the British blockade the Gödels were no longer able to cross the Atlantic and had to use the Transsiberian Raiload in January 1940. They then traveled by way of Japan on a ship to San Francisco and on to Princeton, where they arrived (at the end of March 1940) two months later.

Gödel would never again return to Europe.

6. PRINCETON, USA

(Music: Elvis Presley: "America, America, God shade his grace on thee / and crown thy good with brotherhood/ from to sea to shining sea!)

(OFF/Female speaker:)
At the end of March 1940, Gödel and his wife Adele arrived in Princeton, New Jersey. Henceforth Gödel was to do only his research there at the Institute for Advanced Study. The object of the institute was to provide ideal working conditions for reputable scientific luminaries. ((For this reason there were no students and few teaching obligations.)) The institute was founded 1930 on the initiative of Abraham Flexner, and financed by two millionaires.

((Princeton is a small verdant town in the American state of New Jersey on the east coast of the USA. It had at the time about 4000 inhabitants. The appearance of Princeton differs very little from many other similar provincial towns with their single-family houses built in colonial style (mainly of wood) an accordance with the predominantly Puritan way of life of its residents. The university, built in neo-gothic style which, in its structural form, as so many American universities, imitated time-honored Cambridge (in England), offers the only variety.))

(End: "America, America")

In Princeton, Gödel was a friend in the society of Albert Einstein and J. Robert Oppenheimer (who directed the construction of the first atomic bomb in Los Alamos and who later on was Director of the institute), furthermore John von Neumann, Hermann Weyl, Oswald Veblen and many other famous scientists.

(In any case it was a very great distinction to be called to this olympus of learning. But Gödel's wife Adele also called it teasingly an institution for old age pensioners.)

From his emigration in 1940 up to his death in 1978, Gödel stayed in Princeton practically all his life. In 1943 he began to abandon his direct work on mathematical(-logical) problems. Certain problems in set theory caused him difficulties, for example, (the independence of the Axiom of Choice and of the Continuum Hypothesis from the other axioms of set theory). ((These problems were not solved until 1963 by Paul Cohen.))

(Music: "Harbour Lights")

(Although Gödel had acquired American citizenship, his heart still remained on the old continent.) In the house in Linden Lane, bought in 1949, Gödel lived alone with his wife Adele up to the end of his life. (With the exception of a short period, when he put up his mother in law, who was confined to a wheel-chair.)

(End: "Harbour Lights")

((He led a very secluded life. On the one hand for health reasons, on the other hand out of shyness.

Linden Lane is in a part of town where immigrant German and Italian workers lived. Gödel moved to this district because of his wife, who wanted to live near other German-speaking emigrants. Compared with the villas of his friends Einstein and Morgenstern, Gödel's house seemed very modest by American standards. With its picnic table in the garden, however, it brought the Viennese atmosphere of Grinzing to Princeton.))

(Interview Hlawka:)
((And in Princeton he walked around as an Austrian and as a Viennese. His coat had the specific Austrian cut. He also had no car. He always traveled by bus into the city and to the library.

He was a Viennese in America, although like many a true Viennese, he was born in Brünn.))

(OFF/Female speaker:)
Even his aged mother had to come to Princeton in order to see him. On her 70th birthday Gödel sent her only a recorded disc:

(Original voice of Kurt Gödel:)
I cannot possibly imagine that you should actually be 70 years old.

((For me you have never grown older than 35, and some of the pictures I have seen of you, and also your handwriting, seem to confirm this.)) But I submit to the stern dictum of the calendar (and therefore we both wish you from our hearts the best for your 70th birthday.) May you enjoy in good health many more years the beautiful city of Vienna and its surroundings.

(OFF/Speaker:)
Among his friends mainly from his days in Vienna, were Oskar Morgenstern, ((who, with his Enterprise Mathematica worked for the Pentagon and earned a lot of money by doing so. Mrs. Morgenstern had already known Gödel in her youth and he was often invited as a guest of the Morgenstern family.))

(Interview 1 Dorothy Morgenstern:)
((Well, he was particularly interested in the politics and laws for our country and government, and I think those were the main things, he talked about. At that time, in the beginning, I was on the League of Women Voters Board, and so he thought I should be an expert at all such things. And I found it a little bit difficult always to manage to answer him.)) He came often. Yes. Had his cup of hot water. But we would pick him up and take him to our house, and he stayed about an hour-and-a-half to two hours. And, I would say he probably came every couple of weeks.

He was interested in the children, and in particular in my son as he got older, because hi is a mathematician.

(OFF/Speaker:)
One of Adele's friends was the wife of a emigree historian from Prague Erich von Kahler. ((Mrs. Lili von Kahler, who had given Einstein a famous sweater.))

(Interview 1 Lili von Kahler:)
My first real memory is that I visited Professor Gödel when he was sick with bleeding ulcers in Princeton Hospital together with the Viennese writer Hermann Broch, who had been living with us between 1942 and 1948.

(OFF/Speaker:)
Six years after his ulcer Gödel be naturalized an American in 1948.
((Gödel was interested but not involved in politics.))

(Interview 2 Dorothy Morgenstern:)
((First of all, four weeks before Gödel had called frequently with questions about this aspect of the town and the country and the laws of the United States and so on. So he was very nervous. He wanted to get every detail down.)) So my husband picked him up, and then they picked Einstein up. And when Einstein got into the car, he said: "Now, Gödel, are you ready for your next-to-last examination?" And Gödel said: "What do you mean next-to last? What is my last, what's the last one?" And Einstein said: "When you step into your grave!" So anyway, he got very nervous, and they all went to Trenton. And because of Einstein there was much excitement. Of course nobody had heard of Gödel down there. And Judge (Phillip) Forman took them first and said: "Well Professor Gödel, in your country, in Austria is a dictatorship." And Gödel said: "Yes." And then Judge Forman said: "That couldn't happen in the United States. Could it?" And Gödel said: "Oh yes, it could. And I can prove it!" And Judge Forman said: "Never mind, never mind."

(Music: "Star-Spangled Banner")

(OFF/Speaker:)
How right Gödel actually was, is shown by the McCarthy era, when the rules of democracy were so blatantly violated that it seemed like a dictatorship.

(Interview 3 Dorothy Morgenstern:)
He didn't have a good feeling toward Austria. I mean, he knew, what everybody knew, that many of the people were Nazis, even before Hitler came. And he knew what their sentiments were. And I think he just did not feel that he wanted to go back, or probably even needed to go back.

(OFF/Speaker:)
Surprisingly late in 1953, Gödel was appointed a professor at the Institute, and he received a higher salary.

(Interview 1 Deane Montgomery:)
Well, he was a very conscientious faculty member and he took a great interest in faculty affairs, contrary to what had been presumed by some people in advance.

((He felt very conscientious about it. He was especially interested, as we were, in appointments, partly temporary, but especially in the case of permanent appointments to the Institute.))

(Interview 1 Hassler Whitney:)
Gödel's difficulty ((in deciding about different candidates, what their qualification is, was and should be for a membership)) portended to hold up the faculty meetings. So we desided to shift and have a separate committee on logic. And I offered to take part at that committee. So Gödel and I, we had to communicate mostly over the telephone, He found it easier just to speak to me on the telephone about matters, than first to get together, even if you are right there at the institute.

(OFF/Female speaker:)
((Even in private Gödel preferred communication by telephone, often for hours across the whole continent.))

(Interview 2 Hassler Whitney:)
((... and suggesting I came over, so he did invite me. And then I was there. When I spoke to Gödel directly he did say: "Oh, just tell me over the telephone. We can understand each other over the phone." And occasionally I had said: "This is a matter I feel that's too deep. I Can't talk about it on the phone." And he invited me to come over and talk with him directly. He may feel safer being on a phone, feeling, perhaps, he could cut it off, if he felt too tired. I don't know. If I was there on top of him.)) Maybe he felt anybody too close to him would be on top of him and cause him difficulties.

(Interview 1 Hao Wang:)
((He was what one usually called a recluse. But he is very warm towards personal friends. The other one major thing is, he limited his commitments, in every sense of the word. He had very few people who were close to him. But for those people, who were close to him, they were very close.)) He generally avoided public appearance. So from '51 to '78, there is 26 or 27 years he never gave any lecture in public.

(OFF/Female speaker:)
((Among his colleagues Gödel had friendly relations only to Abraham Robinson and Albert Einstein; with Einstein he went on extensed walks.))

(Interview 2 Deane Montgomery:)
((Gödel ordinarily went to his house at about 10 or 11 in the morning and they walked out here together down this area directly in front of where I am standing and stayed here until about one or two in the afternoon, when I had seen them walking home together. This went on for several years.))

(OFF/Speaker:)
((While Gödel's results from logic entered the classical canon of text-books, he changed the emphasis of his research from logic to physics and philosophy. His former mentor Karl Menger therefore thought that Gödel's genius at logic was wasted in Princeton, as he had no stimulating discussions.))

((Only Einstein arose his old interest in physics again, and subsequently in 1949 Gödel gave some talks on the theory of relativity and started to develop his own cosmological theories. But above all, he also attempted to investigate the philosophical implications of his logic results.))

((Essays on philosophy by Bertrand Russel, Albert Einstein and Rudolf Carnap were written, although not all published. These essays have today become mainstays of the philosophy of mathematics.))

(Interview 2 Hao Wang:)
((He always tried to understand the alternative views, even when the views were different, opposite to his views. And then ho often had to argue, to present his work in such a form, no matter what position you had. That I think is a very good quality. I believe his views have been rather consistent. His favorite philosopher was Leibniz. And he thinks his own philosophy is very close to Leibniz' monodology. There are so many different senses of Platonism. In fact, according to Gödel, Platonism, or Plato himself, only gave the weakest form, everyone has to accept ...))

(OFF/Female speaker:)
((Throughout his life Gödel was opposed to the dogmatism of the Catholic Church; however he adhered to a certain form of private religion and affirmed a logical conception of God. Since Immanuel Kant it was accepted that there is no personal proof of God. Gödel however,

influenced by the works of Charles Hartshorne, constructed a proof of the existence of a logical God. Though critics remarked that the proof even remains true if one replaces the predicate God "G" with the name Gödel!))

(OFF/Female speaker:)
Nor was he at the symposium in Ohio for his 60th birthday.

(Interview 2 Lili von Kahler:)
He was an exceedingly charming, lovely, good man, though he of course had a great deal of personal difficulties. And the relationship was also not so easy, (was it?). And therefore both of them went to a psychoanalyst, to Hulbeck/Hülsenbeck in New York.

(OFF/Speaker:)
Dr. Hulbeck became famous by the name Hülsenbeck as a member of the Zurich group of dadaists. ((Even in New York he maintained contact with his artist-friends Hans Richter and Marcel Duchamp.))

(Interview 1 Rampona:)
Well up to this time his wife was always, they were always shouting at each other. And she said, I took him out of a mental institution, I married him. ((He was a very quiet man. I never heard him say a loud word. After they raised his position / salary at the Institute, he got more money and that was the point where the quarrels with his wife stopped.))

(Interview 3 Hassler Whitney:)
It was in his last period, when his wife was also rather sickly that time and was not in the best of health, in the best mental condition. She would say: "I never met anybody of the Institute. And nobody ever came out to call on me." And Gödel would say to her: "Yes, I remember for example, this person you were talking to, this faculty member." And she said: "Oh no, I never met anybody." And he would listen to her words and find the words were inaccurate. But she was expressing her feelings about it. And he could not listen to her feelings. And neither of them understood, what this problem was.))

(OFF/Speaker:)
((The wife of the longstanding Gödel-friend, the philosopher of science, Paul Oppenheim believes:))

(Interview Gabriele Oppenheim:)
((He had in a way a persecution-complex. And Frau Gödel I also knew of course. She was a very simple person.))

(Interview 3 Lili von Kahler:)
But she was an extraordinarily intelligent person and had an extremely important role, because she was actually, what one calls the life-line. She earthed him. Without her, he could not have existed at all. She once told me: "I have to hold him like a baby."

(OFF/Speaker:)
((In America, too, Gödel had longer periods of illness. As already mentioned, in 1945 on account of an ulcer of the duodenum, he received a blood transfusion and was artificially nourished. Doctor Rampona, who was Einstein's family doctor as well as attending to Gödel from 1936 - 1973, was the only doctor in Princeton who was prepared to provide information.))

(Interview 2 Rampona:)
((Well, he was a rather difficult patient to handle. When I had him in a hospital, I was called to the house one day he was throwing up blood. And he said: "What do you think I have?" I said: "You have a bleeding ulcer!" And he said: "Well, I don't think so.", "Well, what do you think you have?" He said: "I dont know. What are you going to do about it?", "Put you in a hospital!", "I won't go!" Well, then we had to get Einstein to come and tell him to go to the hospital. The reason for his ulcer was the fight with his wife. Aggravation with his wife, that's right.))

(Interview 4 Lili von Kahler:)
((And from then on Gödel took care of his diet. He had a watch with an alarm clock and took every 2 hours his medicaments punctually. He always cooked for himself. Not even Adele who was a good cook was allowed to cook for him. He liked his diet but he also was paranoic and thought that somebody wants to poison him. And so he did until his end.))

(OFF/Speaker:)
((Already at a relatively early point in time, Gödel detached himself from the activities of the institute. On account of this he subsequently had feelings of guilt.))

(Interview 4 Hassler Whitney:)
((There was a period about four of five years before that time, when he was expressing fear about what people thought of him. So I went out to call on him and asked him more about what his fears were. And I gathered that people mostly would say to him: "Oh no, you have nothing to fear. Nobody is going to try to poison you. - Certainly not." And nothing and that. And that would not belay his fears. But his fears came from another direction. I tried asking him about just what his fears were. And he said: "Well, I am not doing the work I ought to at the institute. They expect me as a professor to be doing more things, and take more care of members.))

(OFF/Female speaker:)
((During the last years of his life, Gödel's state of health deteriorated rapidly. He appeared very rarely in public, but privately he cultivated and extended contact with the mathematician and philosopher Hao Wang.))

(Interview 3 Hao Wang.)
((Beginning October '71 we have for about one year notes of this. I think I am mostly struck by the fact that he made (his life) with very little commitments. He only wanted to do a very few things. But everything should be done well. Another very strange thing about him is that in his Nachlass among the most interesting things are many letters he wrote, but did not send. He

wrote very careful replies to people's inquiries. But he then decided not to send the replies. But these are all kept in his files.))

((Gödel was always very strongly for peace. And he detested unnecessary conflicts.))

(OFF/Speaker:)
((Towards the end of his life Gödel occupied himself with occultism. Gödel's life, his reserve, his intellectual style, were since his youth marked by the longing for a purely spiritual, almost immaterial life, analogous to Buddhism. In his library one can find books by Arthur Koestler, who also wrote about Indian philosophy and Yoga. In one of the books Samadi is described as the final aim of Yoga. According to Arthur Koestler Samadi is characterized in the following way:))

(OFF/Female voice:)
((In a physiological respect, it is a reduction of all the functions of the body, such as heartbeat, pulse, breathing and nutrition. In a spiritual respect, one says that Samadi consists in pure consciousness. Consciousness without design or content beyond consciousness itself. There is, however, also a last, deliberately undertaken Samadi. This causes the death of the body, and of the ego which is tied to the body.))

(OFF/Speaker:)
((Thus Arthur Koestler. But also a psychoanalytical interpretation is possible: Gödel's mother-fixation is articulated in his denial of reality, in his constant retreat to an abstract world of ideas in mathematics. His Platonism is the psychological expression of his uterine setting-aside of reality, from which he wanted to withdraw as far as possible.))

((This is made clear for example by his strong wish for communication by telephone instead of personal dialogue. Nevertheless, reality did catch up with Gödel: Since his exile to America, he had a feeling of failure and lack of success. His persecution-mania transposed this feeling, and in an act of pseudo-rationalization and suppression, he shifted the blame on to others, for example on to the Institute, or to Nazi-Austria, which had driven him into exile.))

(OFF/Speaker:)
In the last period of his life Gödel got a persecution mania, as acquaintances testified.

(Background music: Dies irae, dies illae)

For 20 years he had taken hardly any food out of fear of getting poisoned. Kurt Gödel died on 14th January 1978 in Princeton hospital for refusal of food. ((Except Dr. Rampona, all attending doctors refused any kind of information on this subject.))

(End: Dies irae)

(Interview 3 Dr. Rampona:)
Malnutrition. He refused to eat. He never weighed very much, but his final weight was around 68 pounds. And he died in the foetal position. With your knees drawn up. The same position you are in, when you are in your mother's womb.

(OFF/Female speaker:)
As if he never wanted to leave his mother's body's protection. In that way Gödel's life is marked by the longing for a purely spiritual (almost immaterial) existence since his early youth. Gödel's mother-fixation expresses itself in his continuos regression from reality to the abstract world of Platonic ideas.

((The mother-fixation also created an ambivalent structure, the paradox of the double-bind, namely idolizing the mother on one hand, and wishing for her death on the other hand. That is why, since his earliest youth, he admired older women, on the one hand as substitute for his mother, on the other hand in order to obliterate her. This he could do best by marrying an older woman who was hated by the mother.))

(Interview 5 Lili von Kahler:)
((It was great luck that he died before her. He was absolutely desperated, when she was so ill. One day he said: "Please come to visit my wife!"))

(Music: Wheel of Fortune)

In Princeton, Gödel's main interest was the philosophy of mathematics, also Leibniz, Kant and Husserl. And with them the circle of our biographical characterization closes.
For us Gödel lives on in his works and ideas, which we will explain in the remaining.

7. INFORMATICS AND ARTIFICIAL INTELLIGENCE

(Electronic music of the spheres:)

(OFF/Speaker:)
Although Gödel's proof originally arose from Mathematical Logic, it has a central significance for theoretical computer science. The analogous problem there is the Halting Problem for Turing Machines. In practice Turing machines become computers.

(End: Electronic music)

The Halting Problem is essentially this: Can man formulate a computer program which can test an arbitrary program for its correctness? This program is not permitted to run in an endless loop namely without halting.

The answer to the halting problem is No. Such a computer program cannot exist. Thus a fundamental limit of using programming techniques to solve problems is shown to exist and at the same time, Gödel's proof which means

(Computer voice:)
... the limits of the formal provability of true mathematical sentences ...

(OFF/Speaker:)
... are demonstrated.
This problem of provability is, however, directly connected with the question of whether human thought functions like a machine. This seems plausible to many people because they agree with Turing that (in principle) the human brain functions like a digital computer. (Thought outside of the brain is hard for the natural sciences to imagine, but it was conjectured by Gödel.)

Alan Turing transferred Gödel's initial problem stated for formal (language) systems to one for computers, that is from provability to computability.

Besides his achievements in Marathon races, in mathematical logic and in machine intelligence, he was known particularly for his cracking the German secret code ENIGMA in World War II. Nevertheless in 1954 English society drove him to commit suicide because of his homosexuality. His works concerning computable numbers and his model of computation, long before genuine computers existed, belong to the pioneering achievements in the field of computer science.

(Interview Gandy:)
... and then proved that what the human being can do following a routine can be done by what is now called a Turing machine. That is a very simple device. It just moves backwards and forwards on a tape, marking symbols and then moving to the left or to the right. And what a human being can do can be done by such a machine. At that stage there was no question of turning this into practice. But during the war and with the development of electronics, that became possible. And Turing's idea certainly influenced the design of the first really general purpose computers, especially the work of John von Neumann and Turing himself after the war.

Gödel had shown that a particular form of this problem (about what can be done by routines) could not be done by routines. And I think that was the starting point that suggested to Turing one should be able to characterize what can be done and then to show that there are these things that can't be done, so that is a first very important point.

(OFF/Speaker:)
There are additional reasons why we can designate Kurt Gödel along with Alan Turing and John von Neumann, the constructors of the first electronically controlled programmable computers as one of the forefathers of Artificial Intelligence. More than 50 years ago, Gödel had done for the first time what today is entirely routine for every program: to translate real-

world problems that are formulated in ordinary language, into numbers. Today this coding procedure is called Gödelization.

As was suggested by Herbrand at the beginning of a letter to Gödel, he explicitly defined in 1934 for the first time the so-called recursive functions in mathematics. The recursive functions are those which are composed from quite elementary operations, like, for example, addition or multiplication, and which, by systematic feedback from already calculated function values, eventually arrive at their result.

Today these functions play a fundamental role in computer science. Every programmer knows what the recursive calling up of a procedure is, namely the looping of a subroutine. Furthermore recursive functions led, together with the Lamda-calculus developed by Alonzo Church, to the development of the programming language LISP, which is today the most important language of Artificial Intelligence.

In building the new super-computers of the fifth generation, the Japanese have chosen the programming language PROLOG as a basis. PROLOG is an abbreviation for "Programming in Logic", that means formal predicate logic itself is used as the programming language (whereas the classical programming languages in general execute only commands and procedures.)

In private circles Gödel had repeatedly called for the application of formal predicate logic as a programming language, at a time when no computer scientist could imagine such a thing to work at all.

(Interview Wang:)
((On the matter of computer he believes (at least he wanted to prove) that the human mind is much more than a computer. That he always held that view all through his later life. Of course, in Gödel's thinking, the fundamental ideas in philosophy are not much affected by these trivial advances in technology. Therefore, even though he knew what was going on, I don't think he thought it was valuable to his basic ideas. And I agree with him.))

8. TURING MACHINES

(Electronic music:)

(OFF/Speaker:)
We can best understand Gödel's Proof by studying Turing Machines. One can most easily conceive of the Turing machine as a tape recorder which, however, does not reproduce music,

(End: electronic music)

but read or print figures from a tape. ((This activity represents a calculation procedure.)) After the machine halts, an inscription is left on the tape which represents the result of a calculation.

This happens like this: The Turing machine has a certain internal state and the head reads the sign on the tape at the location immediately in front of it. This pair consisting of the internal state and the external sign form the precondition for the command.

The Turing machine can execute for various different preconditions the corresponding command and supplies a result. A command is clearly a mechanical or electronic instruction which is executed. Such a command looks like this for example: if the Turing machine is in state Z and is just reading the symbol zero, then it should proceed to state S and print the sign 1. The fourth item here, namely printing a digit as an output, can also be substituted for, by making the tape move a space to the left or a space to the right. But the essential thing for such a command, in order to calculate with it, is that it consists of four items: It is a quadruple. Every quadruple in a Turing command consists of two pairs: The first is the precondition, the second supplies the result. Both pairs consist of a state and a symbol. The set of all the commands we can arrange in a list.

A command could be formed look like this:
From the state Red and input Zero transfer to state Orange and output 9.
Another command would be:
From state Z and Input E transfer to the state S and output A,
and so on, and so forth.

The list of all commands of a Turing machine is called its machine table. (Each machine table characterizes a particular Turing machine, so that) for the Turing machine to be able to compute accurately, it is enough to know its machine table.

Now by Gödelization of machine tables, we may in turn assign Gödel numbers to the tables themselves, each table having its own private number. But we can set up an even simpler code by encoding a machine table by a list of numbers in the following way: to each command of a given table we assign precisely the number which can be computed in the way displayed. If we now write these numbers (corresponding to the commands) one right after the other in a row without gaps ((just as we wrote the commands of the machine table one beneath the other)), we get a super-long number which is now declared to be the code of the entire Turing machine.

We can think of this number as also being printed on the tape of a special new Turing machine, which we want to call the "Halt-Testing Machine", HPM for short. It is supposed to test if the other Turing machine with the code 30797576 etc. really always halts for every string of input symbols, no matter what they are. To do this, we write the code on the tape of the Halt-Testing Machine and start it. The number which then remains on the tape is the calculated result.

The halting problem can now be formulated like this: Is there an Universal Halt-Testing Machine which can check every last Turing machine to see if it always stops or not? Its code could be, say (because it's easy to memorize), 1234567890. If this code is now printed on the tape and the machine reads its own code, then we have the case where the universal HPM does not stop, even though we can easily demonstrate it has to stop because of the way we defined it.

Because the universal HPM has these contradictory properties (namely a machine which has to stop but cannot stop), clearly it cannot exist.

This case of self-application in machines is comparable to the self-referentiality of ordinary language. Thus the halting problem is reduced to the problem of finding whether machines will halt when reading their own code numbers or whether they run forever.
If we translate the halting problem into the language of Predicate Logic, then we get an arithmetical statement (which is neither provable nor refutable), and thus we see, that the truth of such a statement cannot be decided with the machinery of mathematical logic.

9. MATHEMATICAL LOGIC

(Mephisto speaks in a scene from Faust:)
"(Waste not your time: time's flight is fabulous
Yet method teaches you to save it); thus,
I counsel first the depths you plumb
Of our Collegium Logicum.
(Its vigour will confine your mind
Like Inquisition boots, you'll find
And teach it hence to walk with reason
Smoothly trained to thoughts in season
Not let it stray through thick and thin
Like Jack-o'-Lanterns without discipline.)"

(OFF/Speaker:)
The original demonstration of Gödel's Proof was not carried out with the help of Turing machines, but with Mathematical Logic. By doing so Gödel had further developed and transformed logic itself, and indeed to such an extent that John von Neumann had this to say when Gödel was presented with the Einstein Award (in 1951).

(OFF/Deep voice, with echo:)
"His achievement ((in modern logic)) is a landmark which will remain visible far in space and time. The subject of logic will never again be the same."

(OFF/Speaker:)
Aristoteles early on knew about the self-referentiality of language and its dangers, for example in the Antinomy of the Liar.

Epimenides, who was a Cretan, said: "All Cretans are liars." This paradoxical situation leads to a real contradiction if we rephrase it to say: "This sentence the I am now saying is false."

Leibniz, the inventor of the infiniesimal calculus, wanted to reconstruct (the Aristotelian) logic with a Calculus Universalis along the lines of arithmetic as a formal system of proof. To this

end he introduced a binary number system with the digits 0 and 1 in order to represent concepts by numbers: an anticipation of the later Gödelization.

The Englishman, George Boole, in search of the Leibnizian "alphabet of human thoughts" published "An Investigation of the Laws of Thought", where he interpreted symbol manipulation as rules for computing with 0 and 1. The numeral 0 corresponds to the falsehood of a state of affairs from a logical standpoint, or the lack of flow of current in a machine circuit. The numeral 1 signifies truth and the flow of current.

Wittgenstein applied this concept of logical atomism to natural language, where the subsidiary sentences are connected by logical connectives (such as conjunction or disjunction). The truth or falsehood of the component sentences determine (by computation rules) the truth or falsehood of the whole compound sentence.

In Propositional Logic ((such rules for computing truth values are expressed by)) truth tables; specify procedures which calculate from the truth values of the parts of a compound statement the truth value of the whole statement.

The connective AND has the following truth table, which we shall explain with the example "The ocean is blue AND the sun is shining".

The whole sentence is always true just in case both of the parts are true. That is the definition of the connective AND. Specifically, the truth tabel distinguishes four possible cases which look like this.

In this way one can also formally explicate the other connectives

NOT (negation)
OR (disjunction)
IF - THEN (conditional).

Boolean Algebra provided the logical design for the integrated circuits of modern digital computers and led to switching algebra. It was further developed by Charles Saunders Peirce, Ernst Schröder and others, including Gottlob Frege, who explicitly appealed to Leibniz:

(Speaker with echo:)
((I had not intended to represent an abstract logic in formulas like those of Boole.)) In fact, I did not want to create a mere "Calculus Ratiocinator" but a "Lingua Characteristica" in the sense of Leibniz.

(OFF/Speaker:)
In carrying out this program, Frege extended Propositional Logic (with its truth tables) to Predicate Logic. This is characterized by expressing quantity within sentences. Thus elementary expressions for quantity like "all" or "some exist" are called quantifiers, sometimes including numerically more specific ones like "many" or like "none". They are positioned before predicates like "is mortal" or (before other predicates like) "has the property x" etc. Quantifiers,

predicates and connectives form the principle tools of predicate logic in the formalization of mathematics.

In the voluminous work "Principia Mathematica" by (Lord) Bertrand Russell and Alfred North Whitehead, mathematics is entirely derived by analytical inferences from purely logical premises. A purely logical foundation for arithmetic or number theory was aimed at.

Gödel refers in the title of his work "On formally Undecidable Sentences of Principia Mathematica and Related Systems" to this same work by Russell and he proceeded to show that there exist true but undemonstrable sentences in it.

(Window pane clinks and shatters.)

Subsequently it was attempted to substitute formal provability with the notion of computability by machine; (the new concepts of) Turing Machine and computers being the leading ideas. Once this was done, the next step was to show the Halting Problem for the Turing Machine to be unsolvable, in close and logy to Gödel's Proof as well as to Church's Undecidability Theorem.

10. GÖDEL'S PROOF

(Electronic Music, briefly)

(OFF/Speaker:)
Gödel's Proof concerns a conceptual shift from truth to provability. Gödel constructed a number theoretical formula which says the following: "The formula which is written here is not provable."

To construct this formula, Gödel used Gödelization:

to each connective,
quantifier,
variable and
predicate

we assign, in a precise algorithmic procedure, exactly one number. And the same is done to formulas and to whole proofs.

Suppose the unprovable formula we are now dealing with has the Gödel number 1234567890. With the help of self-reference, we construct this sentence: "The formula with the Gödel number 9876543210 is not provable." By the trick of selfreference, we assure that the entire sentence we have constructed has just the Gödel number it refers to inside itself. (Thus it says that it itself is unprovable.)

(OFF/Female speaker:)
Of course the predicates "Proof" of "Gödel number" do not occur in arithmetic. Also the sentence we are dealing with of course does not really belong to arithmetic, but is merely the abbreviation of a very long formula by numerals, functions and quantifiers.

(OFF/Speaker:)
Only by interpreting this formula with this Gödelization does it obtain the meaning that it itself is not derivable within formal Peano-Arithmetic.

... But nevertheless this sentence is indeed true, because all of its instances are individually true. (Some parts are simple, and therefore also provable.) Others could be very complicated.

The Situation is like the Generalized Party Problem, which is also true but not formally provable. Similarly we could think of Gödel's arithmetic formula as representing the genetic code of the brain, with which the brain itself could examine to check whether it will make mistakes or not, as in the case of the universal Halt-Checking Machine.

(<<To this end, Russell's Antinomy, named after Lord Bertrand Russell, can be applied to Turing Machines. Russell formulated his famous Antinomy in terms of a barber in an English village who signed a constract with the mayor "to shave all inhabitants of the village who do not shave themselves and no one else!" At the end of the year the mayor refuses to pay the barber and argues that he had not filled the contract: He was not allowed to shave the barber, because the barber, as inhabitant of the village, had shaven himself. But the contract says: "Shave exactly all those who don't shave themselves." Since the barber shaved himself, he violated the contract.

"Damn it", cursed the figaro, "that will not happen to me again!" He engaged the haircutter of the neighbouring village to shave him in the future. But at the end of the new year the mayor again refuses to pay. Why is this? The barber did not shave himself. By the contract however, he had "to shave all inhabitants who do not shave themselves". Thus he had to shave himself and he did not do so!

What should the barber do? He has to sign a new contract because this one is unsatisfiable in principle. Russell's Antinomy is no game with words, but a very difficult problem of Set Theory. The question there is the following: Is the collection of all sets which do not contain themselves as elements a set. You can display that collection as a formula (as you can see on the screen) and ask if it contains itself as an element. You can see the contradiction even optically: If it contains itself, by definition it should not contain itself at the same time, and so the converse is also true. Thus a Russell-set cannot really exist!>>)

(OFF/Female speaker:)
Because the unsolvability of the Halting Problem is a computerized version of the undecidability of the Problem of Truth, we will now prove, for the sake of simplicity but quite in the spirit of Gödel's proof, that a universal Halt-Testing Machine cannot exist:

To do this, we shall define Egoists as machines which recognize their own code-names and then halt. But the Altruists are machines which never come to a stop when reading their own code-names, but when reading a different number.

In addition there is a Halt-Testing Machine called CHECKER, and this recognizes all Egoists by halting whenever it reads the code-name of an Egoist; but if CHECKER reads the code-name of an Altruist, then it never stops.

And second, and more important, the Halt-Testing Machine which recognizes Altruists we call EXPERIMENTER. Whenever EXPERIMENTER reads the code-name of an Altruist, he has to stop. But if EXPERIMENTER reads an Egoist's code, it runs on forever.

Suppose we take any Turing Machine at random and we want to see if it is an Egoist or an Altruist, ((that is whether or not it halts at reading its own code-name)), we simply feed that Turing Machine's code-name into CHECKER and simultanously into EXPERIMENTER. One of the two must halt in any case. ((The Siamese twin consisting of CHECKER and EXPERIMENTER together is thus the Universal Halt-Testing Machine we were looking for.))

In case the Turing Machine we are tetsing is an Egoist, CHECKER will halt when seeing that machine's code-name (and thereby recognize its Egoism), whereas EXPERIMENTER runs on past everything (because it only halts for Altruists).

In case the Turing Machine we are checking is an Altruist. EXPERIMENTER will halt when reading that machine's code-name (and thereby recognize its Altruism), whereas CHECKER runs on forever (because it only halts for Egoists).

(OFF/Speaker:)
Unfortunately it can be proved that EXPERIMENTER cannot exist!

(1) Assume it is an Egoist? We reproduce a copy of it and place it in the left upper quadrant among the Egoists.
(2) As an Egoist, it would have to halt for its own code-name. That's how Egoists are defined.
(3) But according to its definition, EXPERIMENTER was supposed to ignore all Egoists and continue running!
(4) Its tape will thus halt and not halt for the same code-name, and we have a contradiction.

(5) So assume it is an Altruist. We make another copy of it, although of course it's really the same Turing Machine as before.
(6) Not as an Altruist, it should run on past its own code-name forever.
(7) But EXPERIMENTER was originally defined to halt when reading any Altruist's code.
(8) Thus, it must again feed its tape through forever as well as halting it and so it cannot be an Altruist either.

(OFF/Female speaker:)
Both assumptions therefore lead to a contradiction. Hence an universal Halt-Checking Machine is logically impossible!

(Speaker:)
Reading one's own code-name corresponds in logic to inserting the Gödel number of the formula into itself which states that the formula with the Gödel number dot-dot-dot is unprovable.

By this simple reinterpretation, we obtain a proof against the existence of a formal system which checks whether arbitrary mathematical statements within that formal system are true or false.

(Electronic music, briefly)

11. WINDOW OF THE MIND

(Moussorgsky: "Pictures at an Exhibition": Promenade)

(Weibel/Schimanovich alternate:)
We hope that we made visible through the window of this film Gödel's world and the invisible culture of mathematics at least for these 80 minutes. Maybe there were times you wanted to switch off the TV. Now you can.

(OFF/Female Speaker:)
According to Gödel's ideas in the final analysis there remains an element of intuition in mathematics.

((Interview Hlawka:))
((There exists no machine that can solve every mathematical problem, therefore the mathematician is indispensable.))

(OFF/Female speaker:)
Gödel's proof seems to imply at first glance the inexhaustibility of mathematics. The philosophical relevance was his mathematical motivation.
((Gödel not only came to this result on his own through his precise knowledge of mathematical foundational research, but he was also inspired by the great philosophers, from Leibniz to Kant, whom he began to study when he was fifteen years old.))

His great discovery, the first limitation theorem of modern times ((shook the foundations of mathematics and)) has been regarded as a refutation of 2000 years of wishful thinking about the mechanization of thought. ((With this critique of men's omnipotent fantasy he stands in the tradition of Nicholas Kopernikus, Charles Darwin and Sigmund Freud.))

Later Gödel revised his standpoint and said it is very possible that there are machines that can prove all mathematical truths, only we cannot identify with certainty the machine which does so.

(End Moussorgsky, Start: Barcarole)

In the end, it turns out that the mind can be represented mechanically, only without our realizing it. Gödel gave Artificial Intelligence start an opening and with that he opened wider the windows to the mind.

(Barcarole remains until the end of the epilogue)

EPILOGUE

(Melody: Barcarole continued)

(Scrolled script:)
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Graphics: Helmut Stadlmann (and Marcus Hanzer)
Mach-One-Cut: Hannes Neubauer
Executive Production: Kurt Hofer
Editorial Assistent: Marleen Schimanovich
Production Superintendent: Alfred Payerleitner
Script and Direction: Peter Weibel and Werner Schimanovich

A Production of Austrian Radio and Television, 1986
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